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Some comments on heat-transfer laws for fine wires

By L. J. S. BRADBURY† AND I. P. CASTRO
Aeronautics Department, Imperial College, London

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The semi-empirical heat-transfer laws of Collis & Williams (1959) and Davies & Fisher (1964) give values of the heat-transfer rates for the flow past fine wires which are generally very different from one another. This paper describes some measurements of heat-transfer and convective time constants which show that the relationship of Collis & Williams is the more representative expression.

1. Introduction

During an investigation into a pulsed-wire technique of velocity measurements (Bradbury & Castro 1971) it was necessary to make use of a heat-transfer law for the flow past heated fine wires of the sort used in hot-wire anemometry. Two apparently careful investigations of this heat-transfer law are those of Collis & Williams (1959) and Davies & Fisher (1964). The heat-transfer law proposed by Collis & Williams for the wire Reynolds number range $0{\cdot}02 < R < 44$ (R based on the diameter of the wire) is given by

$$N_m(\theta_m/\theta_a)^{-0\cdot17} = 0{\cdot}24 + 0{\cdot}56R_m^{0\cdot45}, \tag{1}$$

where θ_a is the ambient fluid temperature and θ_m is the mean of the fluid and wire temperatures. The fluid thermal conductivity and the kinematic viscosity appearing in the Nusselt number N_m and the Reynolds number R_m are evaluated at this mean temperature. In contrast to this expression, Davies & Fisher proposed the following relationship:

$$N_h = 0{\cdot}425R_a^{\frac{1}{3}}, \tag{2}$$

where the thermal conductivity appearing in the Nusselt number N_h is evaluated at the wire temperature and the kinematic viscosity appearing in the Reynolds number R_a is evaluated at the ambient fluid temperature. It will be noted that the suffices a, h and m are used to denote quantities evaluated at the ambient fluid temperature, the hot-wire temperature and the mean of the two respectively.

These two laws are generally very different from one another, and in order to illustrate this, figure 1 shows a comparison between the two laws for wires at essentially ambient temperature and at 600 °C. In order to make a direct comparison between the heat-transfer rates predicted by the two laws the Nusselt numbers have been calculated from (1) and (2) and then converted to a common form based on fluid properties evaluated at ambient temperature. A similar adjustment to the Reynolds numbers has also been made. At the lower temperature there is almost a two to one difference in the heat-transfer rates predicted by

† Present address: Mechanical Engineering Department, University of Surrey.

the two laws, but as the wire temperature is increased this difference decreases until the two laws are almost coincident at 600 °C.

Because of the common usage of constant-temperature hot-wire anemometers there is less interest in the precise form of fine wire heat-transfer laws than in the days when constant-current hot-wire anemometry predominated. Nevertheless,

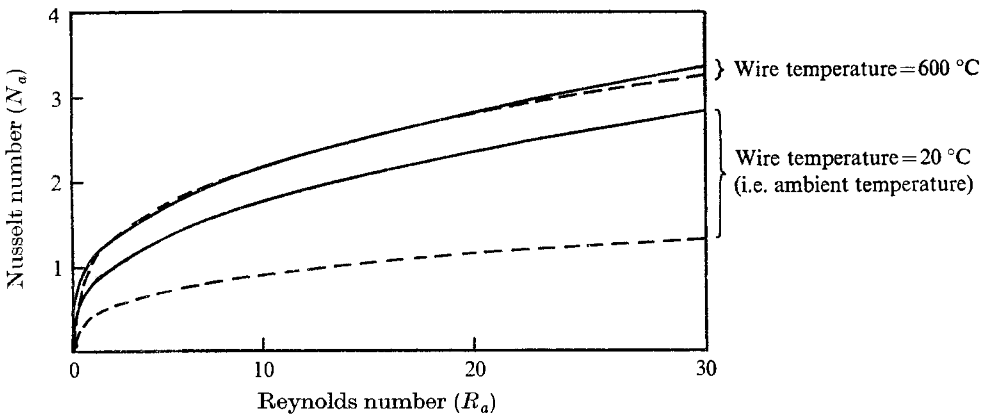

FIGURE 1. Comparison of heat-transfer laws for fine wires. ——, Collis & Williams; ----, Davies & Fisher.

it is obviously important to resolve discrepancies of the sort found between the relationships of Collis & Williams and Davies & Fisher. This was the aim of the experiments described in this paper. It should be made clear that these experiments were not intended as a definitive study of heat-transfer laws, but in view of the size of the discrepancy between equations (1) and (2) it seemed that a few reasonably careful measurements ought to be sufficient to show which of the two laws is the more representative for the heat transfer from fine wires.

The results of the present measurements strongly support relationships of the type suggested by Collis & Williams. The experiments consisted not only of heat-transfer measurements but they also included some direct measurements of wire convective time constants. These latter measurements give some particularly pertinent results for deciding between the relative validity of the two laws.

2. Heat-transfer measurements

The two quantities that have to be evaluated from the experimental results are the wire Reynolds number R and the Nusselt number N. In practical terms the Nusselt number can be written as

$$N = \frac{i^2 r_h}{\pi k_f l} \frac{\alpha r_0}{(r_h - r_a)},$$

where r_h and r_a are the wire resistances at the hot-wire temperature θ_h and the fluid ambient temperature θ_a respectively. α is the temperature coefficient of resistance, i is the current through the wire of length l and diameter d, k_f is the fluid thermal conductivity. There is no disagreement between Davies & Fisher and other experimenters over the ability to measure i, r_h and l accurately;

the nub of their criticism of previous work concerns the measurement of α and r_a. Their contention is that measurements of these quantities are carried out at very low current densities and are likely to be in error by as much as 20 % owing to the influence of thermo-electric effects present in the bridge network used for making the measurements. The effect of this error would be particularly significant at very low overheat ratios, and they claim that heat-transfer measurements at low wire temperatures must be regarded as unreliable. However, no evidence of Peltier effects of the type described in their experiments could be found in the present work.

Nominal diameter (in.)	Measured diameter (in.)	Resistance per centimeter at 20° C. (Sample lengths of about 0·5 cm)	Resistivity at 20 °C (ohm cm)
0·0002	$0{\cdot}000204 \pm 5 \times 10^{-6}$	51·9	10·9
0·0003	$0{\cdot}000315 \pm 10 \times 10^{-6}$	22·5	11·3
0·0004	$0{\cdot}000412 \pm 14 \times 10^{-6}$	13·4	11·5
		Average value	11.2

TABLE 1. Platinum wire

An ordinary Wheatstone bridge was used for the heat-transfer measurements. The variable arm consisted of a high-quality decade resistance box and the fixed arms used wire wound resistors of 0·01 % accuracy. The current through the wires was obtained by measuring the voltage drop across the fixed resistor in series with the hot wire. This voltage was measured on a four decade digital voltmeter with a least significant figure equivalent to 2·5 microvolts on its most accurate range. This voltmeter was also used to balance the bridge.

The wires used in the majority of the experiments were platinum Wollaston wires from Johnson, Matthey & Co. Ltd. The platinum used in Wollaston wire is not pure but apparently contains about 0·3% of iridium and rhodium. Samples of the wires after etching were examined under a scanning electron microscope and a typical example is shown in figure 2 (*a*) (plate 1). An interesting feature of the platinum wires is the longitudinal striations on the surface. Presumably these arise from the drawing process used in their manufacture. Variations in diameter (or roundness) over the length of the samples examined were not very significant although, on the basis of resistance measurements, there were perhaps more consistent variations of about 5 % over longer lengths of the wire. A few measurements were also made with nickel wire† (nominal diameter of 0·0005 in.) and figure 2 (*b*) (plate 1) shows a sample of this wire. This was a sample of wire from an old reel of nickel wire and some oxidation of the surface had clearly taken place.

The 'cold' resistances of the electron microscope samples were measured and table 1 summarizes the results for the platinum wires.

† The object of the nickel wire tests was to check that if the thermo-electric effects were arising from the wire junctions then tests with a different wire material might show some inconsistency.

The wires used in the heat-transfer measurements were mounted on a probe giving a wire length usually of about one centimetre so that aspect ratios ranged from about 1000 to 2000. A number of wires were used in the experiments and table 2 gives the 'cold' resistance per unit length of these wires. These values differ slightly from those from the electron microscope samples given in table 1. This is presumably because of small variations in the wire diameter.

Nominal diameter (in.)	Resistance per centimetre	Estimated mean diameter (in.)
0·0002	49·0	0·00021
0·0003	22·2	0·00032
0·0004	12·2	0·00043

TABLE 2

The temperature coefficient of resistance of the wires was measured by mounting the probes in a test tube immersed in a variable-temperature water bath. The temperature coefficient of resistance was found to be 0·0035 per °C for the platinum wire and 0·0059 per °C for the nickel wire.

A criticism by Davies & Fisher of previous work was that 'cold' resistance measurements were probably in error owing to stray thermo-electric voltages present in the bridge network; they cited an example in their own experiments of how this could give rise to errors, typically of 20 %, in the values of cold resistance measurement. However, in the present experiments measurements of 'cold' resistance levelled off as the current through the wires was reduced and the values did not increase again – as they did in the example quoted by Davis & Fisher – even though the current densities and dissipation levels were extended well below those reported by Davies & Fisher. They also attributed the apparent departure of the wire material properties from the International Standard values as being a result of these thermo-electric effects. However, the influence of iridium and rhodium impurities in platinum is to increase the resistivity and reduce the temperature coefficient. Results given by Darling (1961) and Rhys & Taimsalu (1969) show that the observed differences are of about the right order for an impurity level of 0·3 %. Since no significant thermo-electric effects could be found in the present experiments, it is concluded that the departures found both in the present experiments and those previously carried out by Collis & Williams and others would seem to be genuine, although the evidence is admittedly circumstantial.

The measurements of the Nusselt number were made in a low turbulence stream with wire temperatures ranging from between 20 °C and 600 °C above ambient temperature. Most of the tests were carried out with wires whose aspect ratios were in the range 1000 to 2000. However, two tests were also carried out with wires of lower aspect ratios of about 270 and 800. All the results were corrected for aspect ratio effects using the simple relationship

$$N(\infty) = N(m)\left[1-\frac{d}{l}\left(\frac{k_w}{k_f N(m)}\right)^{\frac{1}{2}}\right],$$

where $N(m)$ is the measured Nusselt number and $N(\infty)$ is the corrected Nusselt number for an infinite wire, k_w is the thermal conductivity of the wire material and k_f is the thermal conductivity of the fluid. This expression can be derived on the assumption that the aspect ratio is large and that the overheating ratio is small. Whilst these conditions were not strictly fulfilled in all the test cases, the object of the tests at the lower aspect ratios was to check the overall consistency of the experimental results.

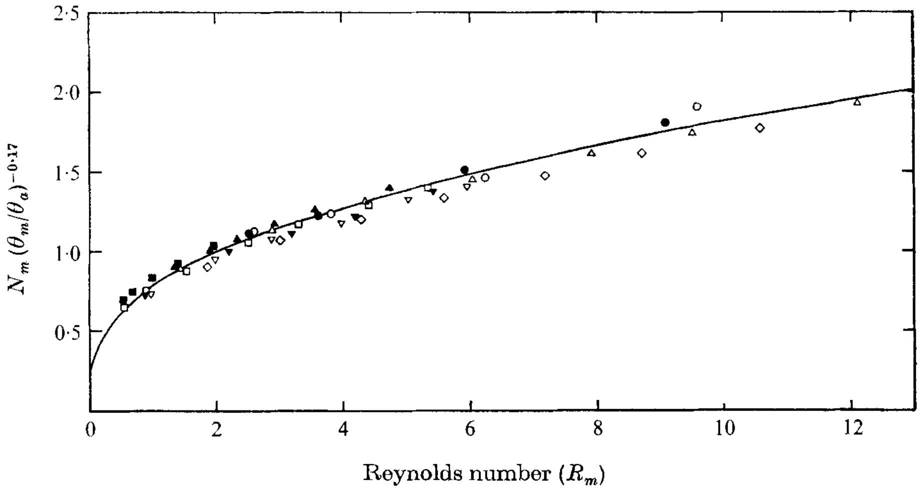

FIGURE 3. Heat-transfer results compared with the Collis & Williams law (solid line). □, ■, 0·0002 in. Pt wire, aspect ratio. 1790, wire temperature 125 °C, 507 °C above ambient respectively; ○, ●, 0·0003 in. Pt wire, aspect ratio 1850, wire temperature 20 °C, 40 °C above ambient respectively; ▽, 0·0003 in. Pt wire, aspect ratio 274, wire temperature 100 °C above ambient; ▼, 0·0004 in. Pt wire, aspect ratio 795, wire temperature 125 °C above ambient; △, ▲, 0·0004 in. Pt wire, aspect ratio 1350, wire temperature 145° C, 615 °C above ambient respectively; ◇, 0·0005 in. Ni wire, aspect ratio 1270, wire temperature 105 °C above ambient.

At the lowest aspect ratio tested, the correction amounted to about 15 % on the Nusselt number whereas the correction for the other results was only a few per cent. However, although the consistency of the final correlation was not greatly affected by the correction, it served to demonstrate that there is no unexpected influence of aspect ratio on the measurements. These few aspect ratio tests were undertaken as a precautionary measure because Davies & Fisher's measurements were all carried out at comparatively low aspect ratios and it seemed expedient to check the possibility of some untoward aspect ratio effect.

The experimental results are shown first in figure 3 plotted in terms of the fluid properties evaluated at the mean temperature. They show good agreement with Collis & Williams' relationship and there is no systematic difference between the high and low overheat ratio results. The nickel wire results are also reasonably consistent with the platinum wire results. In contrast to this, figure 4 shows some selected results at two extreme overheating ratios evaluated first on the mean temperature basis as in figure 3 (unblocked symbols) and then with the thermal conductivity of air evaluated at the wire temperature and the kinematic viscosity evaluated at ambient temperature (blocked symbols). The effect of using the temperature loading suggested by Davies & Fisher is to spread out the experimental points and, as might be expected from figure 1, it is only at temperatures in the region of 600 °C that good agreement with their relationship is found. It seems almost inconceivable that errors due to Peltier effects should be the same in both the present experiments and the work of Collis & Williams, and the

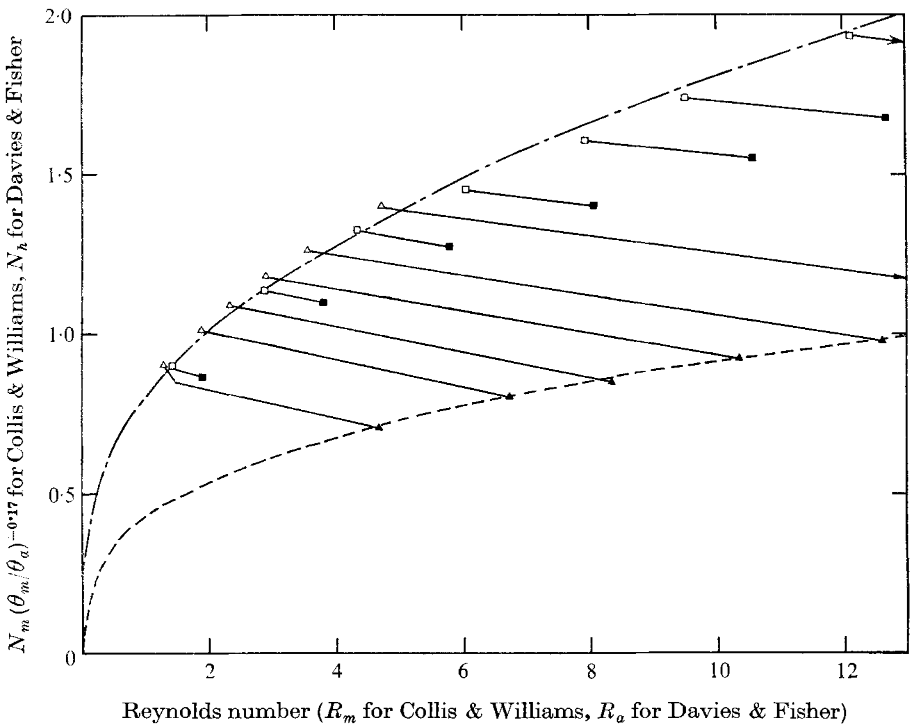

FIGURE 4. The influence of the reference temperature on the heat-transfer results. – – – –, Davies & Fisher; ——, Collis & Williams. 0·0004 in. Pt wire, aspect ratio 1350; □, ■, 145 °C; △, ▲, 615 °C; unblocked symbols based on the mean temperature; blocked symbols based on the wire temperature for k and the ambient temperature for ν.

obvious conclusion is that Davies & Fisher's relationship is not satisfactory for a wide range of temperatures. However, it could be argued that we have only repeated the errors of earlier experiments and for this reason it is pertinent to include some direct measurements of wire convective time constants that have been made.

3. Measurements of the convective time constants

In the pulsed-wire experiments by the authors, the wires were subject to a very short duration voltage pulse and then permitted to cool by forced convection. By operating the wires with a small constant current passing through them it was possible to observe the decay of the wire temperature. If we neglect the possible influence of variations of fluid properties during the decay, this decay should be exponential with a time constant given by $\rho_w c_w d^2/4k_f N$, where ρ_w and c_w are the density and specific heat of the wire material respectively, d is the wire diameter, k_f is the thermal conductivity of the fluid and N is the Nusselt number. In fact, the observed decays were indistinguishable from an exponential decay and a whole series of time constant measurements were made with different diameter wires at two values of the initial wire temperature of about 250 °C

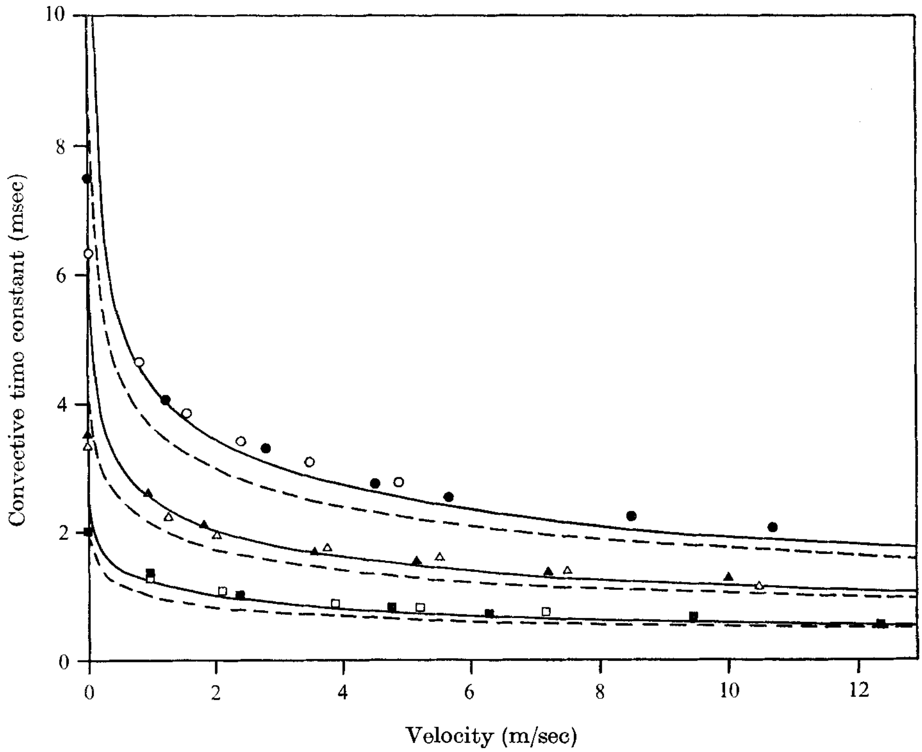

FIGURE 5. Convective time constants for platinum wires. Curves show Collis & Williams' law; ——, ambient mean temperature; ---, 300 °C mean temperature; the upper, middle and lower pairs of curves refer to wires of diameters 0·00043 in., 0·00031 in. and 0·0021 in. respectively. ○, ●, 0·00043 in. diameter; △, ▲, 0·00031 in. diameter; □, ■, 0·00021 in. diameter; unblocked symbols refer to peak temperatures of 550 °C; blocked symbols are results for peak wire temperatures of 250 °C.

and 500 °C. The results of these measurements are shown in figure 5. The interesting feature of these results is that there does not seem to be any measurable influence of the initial temperature on the decay time. This suggests that the product $k_f N$ in the time constant denominator should not depend on temperature. Although neither the relationship of Collis & Williams nor that of Davies & Fisher is entirely consistent with this finding, the Collis & Williams relationship gives values of $k_f N$ which are far less sensitive to wire temperature than those obtained from Davies & Fisher's expression. To illustrate this figure 5 also shows the time constants calculated from Collis & Williams expression for mean wire temperatures of 300 °C and ambient temperature. The ambient temperature results are close to the experimental values but the higher temperature calculations show some influence of temperature on the time constants which one might have expected to be observable in the experimental results. If, however, the time constants are calculated using Davies & Fisher's relationship, there is a much greater influence of the temperature used. To give one example figure 6 shows the measured time constants for the 0·0004 in. diameter wire compared with time constants obtained from Davies & Fisher's relationship for two hot-wire temperatures of 600 °C and essentially ambient temperature. The influence of temperature is far more significant and the decay times differ considerably from those observed. The point about these time constant measurements is that they are not dependent on measurements of cold resistance in the same way as the heat-transfer measurements and they therefore provide separate support for the greater validity of the Collis & Williams relationship. However, the apparent insensitivity of the decay times found in the experimental results suggests that even the Collis & Williams relationship may not necessarily represent the best compromise for an empirical heat-transfer law.

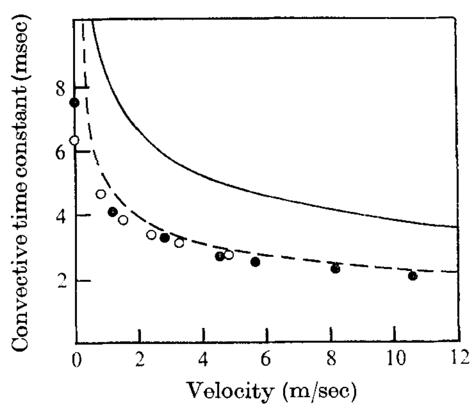

FIGURE 6. Convective time constants for 0·00043 in. diameter platinum wires. ——, Davies & Fisher, wire at ambient temperature; – – – –, Davies & Fisher, wire at 600 °C. Unblocked symbols refer to peak temperatures of 550 °C, blocked symbols are results for peak temperatures of 250 °C.

4. Concluding remarks

There is a considerable discrepancy between the heat-transfer relationship proposed by Collis & Williams (1959) and Davies & Fisher (1964). On the basis of the present experiments it would appear that the Collis & Williams relationship is the more representative expression for a wide range of wire temperatures. However, the wire convective time constant measurements would appear to show that a reference temperature more heavily weighted towards ambient temperature would be more appropriate for collapsing heat-transfer results than the mean temperature.

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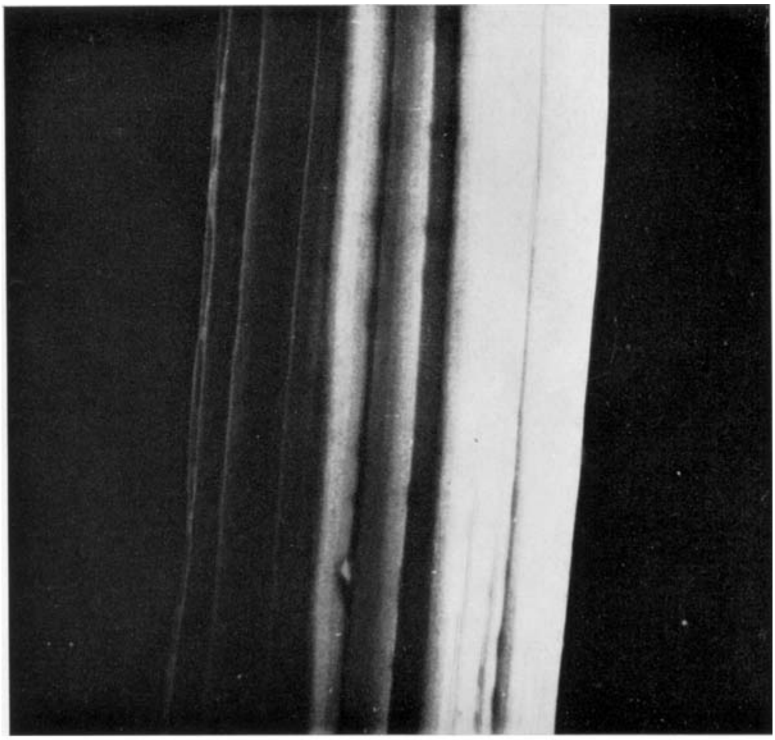
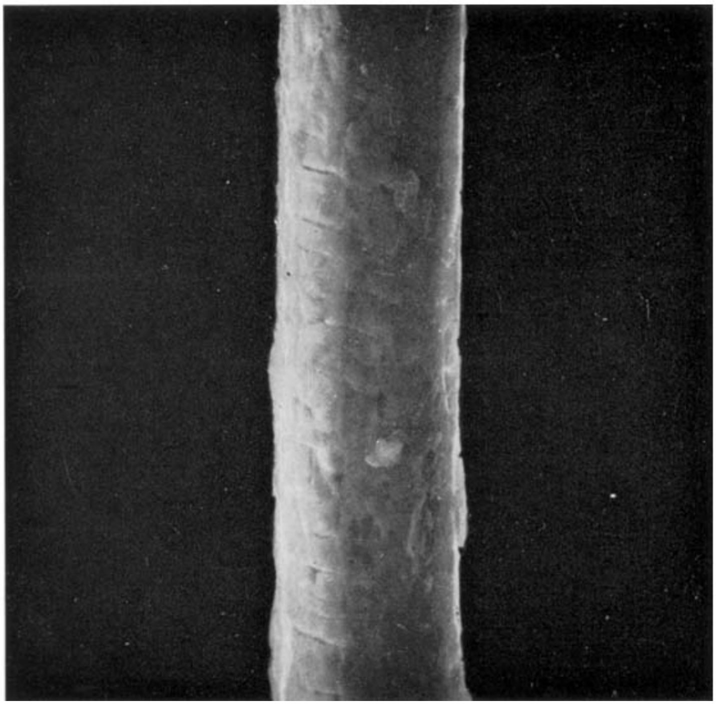


(*a*)



(*b*)

FIGURE 2. Electron microscope photographs of the wires. (*a*) 0·0002 in. diameter platinum, (*b*) 0·0005 in. diameter nickel.